Monitoring Budget Deficits
A Time Series Model for India

Srinivasa Madhur
Wilima Wadhwa

This paper addresses some important issues arising out of the finance ministry's 'Technical Note on Monitoring Budget Deficits' relaesed in August 1990. The paper then develops a simple seasonal ARIMA model for monthly budget deficits and subjects it to forecasting tests. It is found that the out of sample forecasts generated from this model are superior to the B-J forecasts presented in the finance ministry's Technical Note. The paper ends by indicating the scope for further work in modelling the budget deficit.

I
Introduction

IN recent years, budget deficits of the central government have been a matter of concern for Indian policy-makers. Large and persistent budget deficits have serious implications not only for fiscal balance but also for monetary and price stability. Recognising this, the government has, of late, been giving top priority to the monitoring of budget deficits with a view to take appropriate advance action whenever there are indications that the budget deficit is likely to go beyond a certain limit, in any given year. Towards this end, in August 1990, the ministry of finance presented a 'Technical Note on Monitoring Budget Deficits' of the government.

The Technical Note considers several methods of forecasting month-end budget deficits. After empirically implementing a few alternative methods, the Note chooses a Box-Jenkins (B-J) autoregressive model as the most preferred method for forecasting month-end deficits. The B-J model is then used to forecast the March-end budget deficit for the current financial year, i e, 1990-91. Certainly, the Note represents a good beginning towards forecasting budget deficits based on scientific time-series forecasting techniques. There are three important aspects of the Technical Note which need further work.

First, the forecasts of the month-end deficits from the B-J model presented in the Note are consistently lower than the actual month-end deficit for all the twelve months of the forecasting horizon, i e, fiscal year 1989-90. This certainly does not speak well for the particular B-J model chosen for forecasting budget deficits in the Note. The forecast errors of a good model should generally be random and not systematic as is the case with the B-J model of the Note.

Secondly, after presenting forecasts from several alternative methods for the twelve months of the fiscal year 1989-90, the Note selects the B-J model (with a particular autoregressive structure) as the most preferred model for forecasting month-end budget deficits. This model selection has not been done on the basis of any objective statistical criteria. Instead, it is done on the basis of the argument that (i) for forecasting, the B-J model is better than all other methods because it takes into account both intra-year and inter-year variations in the month-end budget deficits; (ii) it helps to forecast the year-end deficit on the basis of movements in the month-end deficit in a given year; and (iii) it can also estimate the probability of the actual deficit remaining within the budget estimate or a specified range. Surely, most of the alternative methods of forecasting budget deficits presented in the Note could perform the last two functions mentioned above; the B-J model does not have a comparative edge in these. As for the first argument, the B-J model may be better in accounting for both the intra-year and inter-year variations in the budget deficit. However, the crucial issue here is not whether the B-J model is procedurally superior to the other methods but whether, on an average, it forecasts month-end budget deficits more accurately than the other methods. A simple but common method of assessing the relative accuracy of these alternative forecasts is to compare their forecasting errors, as measured by, say, the root mean squared error of the forecasts. The Note, however, abstains from applying any such objective model selection criteria.

Thirdly, the B-J model of the Note is estimated using the twelfth difference of the monthly deficit as the dependent variable. It is not clear why this is done. Perhaps, such differencing of the monthly deficit is done to obtain stationarity of the series, a property required for estimating univariate time series models. However, there is no mention as to whether the monthly deficit series itself was subject to any stationarity tests and was found to be stationary or not. If the monthly deficit itself is stationary, there does not seem to be any reason for differencing it before estimating the model. Such over-differencing of the series could lead to non-invertibility of the process, thereby, generating inefficient forecasts.

The present paper addresses these three issues arising out of the finance ministry's Note. To anticipate the conclusions of the paper, we find that no differencing of the series on monthly deficits is required for model building since this series satisfies the stationarity condition. The paper then develops a simple seasonal ARIMA model for monthly budget deficits and subjects it to the forecasting tests; it is found that the out of sample forecasts generated from this

TABLE 1: UNIT ROOT TESTS FOR MONTHLY DEFICIT: 4/1984 TO 3/1989 (T=72)

	Dickey-Fuller			Phillips-Perron[1]		
	$t_{\hat{a}}$	$t_{\tilde{a}}$	$Z(\hat{a})$	$Z(t_a)$	$Z(\hat{a})$	$Z(t_{\tilde{a}})$
Model 1	-11.65		-90.40	-12.01		
Model 2	-5.12		-23.30	-4.77		
Model 3		-11.53			-89.85	-11.92
Model 4		-5.07			-23.00	-4.69
Critical values[2] (95 per cent)						
T = 50	-2.93	-3.50	-13.30	-2.93	-19.80	-3.50
T = 100	-2.89	-3.45	-13.70	-2.89	-20.70	-3.45

Model 1: Testing for unit root at lag 1 without a time trend.
Model 2: Testing for unit root at lag 12 without a time trend.
Model 3: Testing for unit root at lag 1 with a time trend.
Model 4: Testing for unit root at lag 12 with a time trend.

Notes: 1 The statistics reported here are for a serial correlation correction of order 4.
2 Our sample size is 72 and we give critical values for both T=50 and T=100. Using the critical values appropriate to the larger sample biases the tests towards rejecting the null.

FIGURE 1: MONTHLY DEIFICT—MAY 1988 TO MARCH 1989

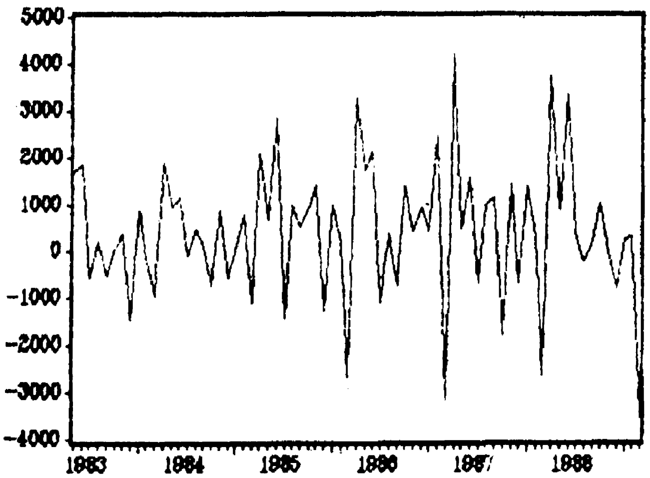
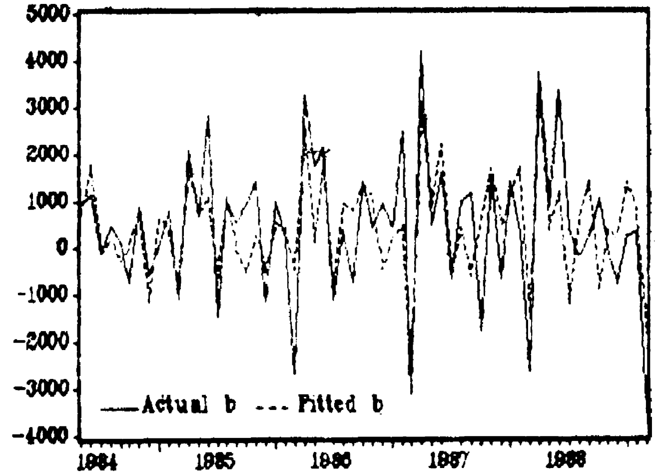


FIGURE 2: IN-SAMPLE TRACKING OF THE MODEL



model are superior to the B-J forecasts presented in the finance ministry's Technical Note. The paper ends by indicating the scope for further work in modelling the budget deficit.

II

Modelling Monthly Deficits: Methodolgy and Empirical Results

Like the finance ministry's Note, we confine ourselves to modelling the behaviour of budget deficit using the B-J type univariate time-series techniques. The series we attempt to model is the 'monthly' budget deficit, which is a flow variable having the dimension of deficit per month. This variable is different from the month-end budget deficits reported in the finance ministry's Note, which is a cumulative sum of monthly deficits, the cumulation starting from April and ending in March within each fiscal year. To avoid possible confusion, whenever notations are used in the rest of the paper, the 'monthly' budget deficit in the t-th month, is denoted by b_t and the corresponding month-end deficit by B_t.[1]

The major steps involved in the univariate :ime-series modelling of the monthly budget leficit are: (i) choosing a method of leseasonalising the monthly deficits; ii) testing for stationarity of the series on nonthly budget deficits and choosing an ap›ropriate technique of filtering the series, in :ase the monthly budget deficit series is 'ound to be non-stationary; (iii) estimating ılternative univariate models of monthly leficits and choosing one of these models or forecasting; and (iv) subjecting the hosen model to an out of sample foreasting test.

DESEASONALISATION

As in the case of most macroeconomic me series variables, the monthly budget eficits exhibit considerable seasonality cross months within each financial year. In ıodelling the temporal behaviour of such series, appropriate allowance must be made for these seasonal effects. One approach, which was commonly used in time series studies of older vintage, is to adjust the raw data on a variable (in this case, monthly budget deficits) for seasonality and then model the temporal behaviour of such a seasonally adjusted series.

More recent developments in time series modelling, however, discourage such prior adjustment for seasonality, as it tends to result in over-adjustment of the series. In other words, seasonal adjustment, more often than not, removes too much power from the spectrum at the seasonal frequencies [see, for example, Harvey 1981 and Granger and Newbold 1977]. This phenomena is reflected in the time domain in terms of a tendency for the seasonally adjusted series to exhibit negative autocorrelations at seasonal lags.[2] For example, Wallis [1974] seasonally adjusted a quarterly white noise series. Ideally, the adjusted series should also have been white noise. However, Wallis found that the adjusted series displayed small positive autocorrelations at lags 1 to 3, 5 to 7,... and somewhat more pronounced negative autocorrelations at lags 4, 8,... which would suggest fitting an AR(4) model to a series which is white noise! The conclusion which emerges from this is that it is better to work with unadjusted data, since seasonal adjustment can produce considerable distortions without guaranteeing a seasonality free series.

However, since it is quite obvious that seasonal effects are present in the monthly deficits, we need a model which will account for both seasonal and non-seasonal movements. We, therefore, adopt the multiplicative seasonal ARIMA model, proposed by Box and Jenkins [1976], which is the same as that used by the finance ministry.

A multiplicative seasonal ARIMA process of order $(p,d,q)X(P,D,Q)_s$ can be represented by

$$(1)\ \phi^s(L^s)\phi(L)\Delta^d\Delta_s^D b_t = A + \theta^s(L^s)\theta(L)\,\varepsilon_t$$

where D and d are integers denoting the number of times the seasonal and the first difference operators are applied, respectively, $\theta^s(L^s)$, $\phi(L)$, $\theta^s(L^s)$, and $\theta(L)$ are polynomials in the lag operator of orders P, p, Q and q, respectively, and A is a constant term.

TESTING FOR STATIONARITY

The next step in modelling the budget deficit is to test whether the monthly deficits are stationary. In case the monthly deficits are found to be non-stationary, an appropriate order of differencing will have to

TABLE 2: ONE-MONTH AHEAD FORECASTS OF MONTH-END DEFICITS

(Rs crore)

Month	Actual	$\hat{B}$[1]	FM[2]
April 89	4628	3725	3914
May 89	6777	4848	4733
June 89	9852	8999	6719
July 89	11390	10149	5966
August 89	12403	10951	6472
September 89	12431	12330	6704
October 89	11151	13499	6821
November 89	13082	12187	7513
December 89	11789	11904	7389
January 90	14504	12457	8144
February 90	13908	13934	9221
March 90	11466	11699	6151

Notes: 1 Month-end deficit forecast from our model.
2 Month-end deficit forecast from B-J model of FM.

Figure 3: One-Month Ahead Forecasts of Month-end Deficits, Rs Crore

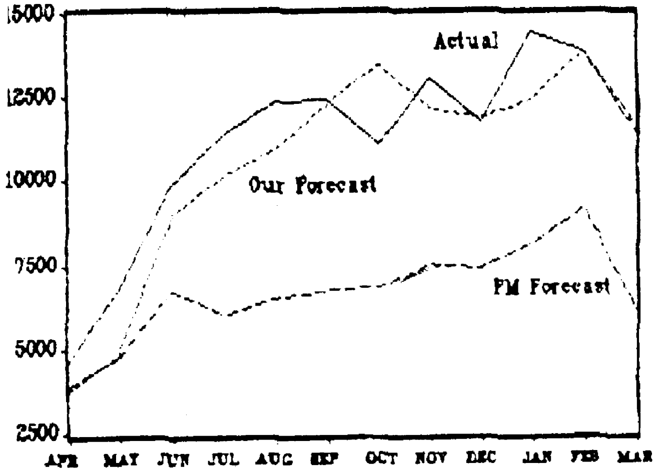
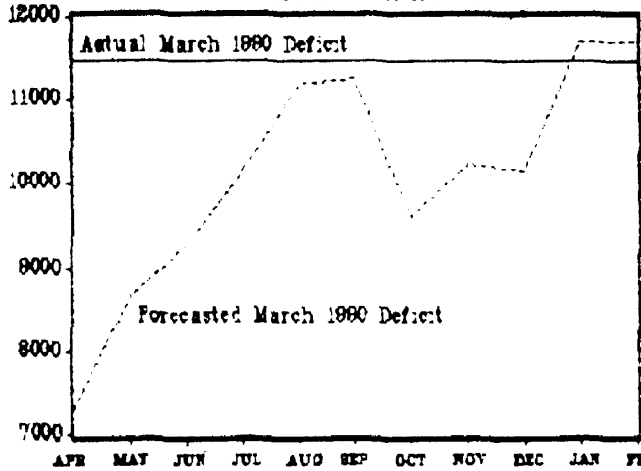


Figure 4: March-end Budget Deficit Forecasts from Various Months of 1989-90



be undertaken so as to yield a stationary series of monthly deficits. Traditionally, econometricians considered the plot of a series against time, to determine if it was stationary. However, recently, more rigorous tests have been developed to test for stationarity of a time series [see, for example, Dickey and Fuller 1979 and 1981, Said and Dickey 1984 and Phillips and Perron 1988]. The new approach, pioneered by Dickey and Fuller, seeks to determine whether a series has a unit root, which is tantamount to saying that it is non-stationary.[3] The approach is very appealing, in that the tests are simple to perform since they are based on statistics from a simple ordinary least squares (OLS) regression.

Figure 1 shows monthly deficits from May 1983 to March 1989.[4] The series looks stationary around a mean of zero. We, nevertheless, subjected it to the various stationarity tests proposed in the literature. We consider two alternative hypotheses: (i) there is a unit root at lag 1; and (ii) a unit root exists at lag 12. For each of these, two variants were investigated, namely, with and without a time trend. Table 1, reports results from the standard Dickey-Fuller test [1979, 1981] and those from the tests proposed by Phillips and Perron [1988]. The Dickey-Fuller test is a parametric test and assumes that errors are NID(O, σ^2). On the other hand, the Phillips-Perron test is non-parametric with respect to nuisance parameters and, thereby, allows for a wide class of time-series models in which there is a unit root.

We base the Dickey-Fuller test on the following OLS regressions:

(2) $(b_t - b_{t-1}) = \hat{\mu} + \hat{\alpha} + b_{t-i} + \hat{\varepsilon}_t$

without a time trend, and

(3) $(b_t - b_{t-i}) = \tilde{\mu} + \tilde{\beta}(t - T/2) + \tilde{\alpha}\, b_{t-i} + \tilde{\varepsilon}_t$

with a time trend, and

$\hat{\alpha} = (\hat{\rho} - 1)$; $\tilde{\alpha} = (\tilde{\rho} - 1)$. $\quad i = 1, 12$

H_o: $= 1$ $\quad t = 1, 2, \ldots, T$

where b_t is monthly deficit. The critical values for the t-statistics of $\hat{\alpha}$ and $\tilde{\alpha}$, $t_{\hat{a}}$ and $t_{\tilde{a}}$ can be obtained from Fuller (1976), Table 8.5.2.

The Phillips-Perron tests are based on the following OLS regressions:

(4) $b_t = \hat{\mu} + \hat{\alpha}\, b_{t-i} + \hat{u}_t$

without a time trend, and

(5) $b_t = \tilde{\mu} + \tilde{\beta}\ (t - T/2) + \tilde{\alpha}\, b_{t-i} + \tilde{u}_t$

with a time trend, and

H_o: $\alpha = 1$ $\quad i = 1, 12$

$\quad t = 1, 2, \ldots, T$

They define transformations of conventional statistics from (4) and (5) above, $Z(\hat{\alpha})$, $Z(\tilde{\alpha})$, $Z(t_{\hat{\alpha}})$ and $Z(t_{\tilde{\alpha}})$, which eliminate the nuisance parameter dependencies asymptotically.[5] The critical values for Z(.), can be fount in Tables 8.5.1 and 8.5.2 of Fuller [1976].

From Table 1, we see that the null hypotheses of a unit root is rejected for all the tests undertaken.[6, 7] We, thus, conclude that monthly deficits are stationary over the time period considered. In other words, no differencing is needed to model the series.

This has important implications for the B-J model given in the finance ministry's Note. In the Note presented by the finance ministry, the twelfth difference of monthly deficits, $(b_t - b_{t-12})$, is modelled, implying that monthly deficits have to be differnced to get stationarity. However, our results indicate that no such differencing is needed, as the series on monthly deficits itself is stationary. The B-J model of the finance ministry's Technical Note, thus, runs the dangers of overdifferencing: Overdifferencing often leads to non-invertibility of the process, which is a problem in terms of identification and one also has to be careful in adopting an appropriate estimation procedure. More importantly, non-invertible processes lead to inefficient forecasts [Harvey 1981].[8] This point is especially important, in view of the fact that forecasting budget deficits is the ultimate goal of the finance ministry's Technical Note.

A Univariate Model of Monthly Budget Deficits

Having found that the monthly deficit series is stationary, the next step in modelling it is to try out alternative specifications and choose one among these as the most preferred version. For this, we examined the

Table 3: Dynamic Forecasts of Month-End Deficits

(Rs crore)

Month	Actual	Apr 89	May 89	Jun 89	Jul 89	Aug 89	Sep 89	Oct 89	Nov 89	Dec 89	Jan 90	Feb 90
May 89	6777	4848										
June 89	9852	7843	8999									
July 89	11390	8172	9638	10149								
August 89	12403	8218	9559	10208	10951							
September 89	12431	8533	9924	10518	11461	12330						
October 89	11151	9486	10857	11472	12335	13438	13499					
November 89	13082	9627	11006	11612	12507	13517	13593	12187				
December 89	11789	9167	10543	11153	12035	13082	13152	11368	11904			
January 90	14504	9501	10878	11486	12374	13406	13479	11846	12526	12457		
February 90	13908	9886	11262	11872	12757	13795	13867	12173	12796	12708	13934	
March 90	11466	7284	8660	9269	10155	11191	11263	9594	10240	10160	11715	11699

autocorrelation and partial autocorrelation structure of the monthly budget deficits for the period May 1984 to March 1989. (See Appendix III for a plot of these). Taking cues from this, we fitted a number of univariate autoregressive-moving average (ARMA) models. The chosen formulation is a simple seasonal ARIMA of order $(1,0,0) \times (1,0,0)_{12}$.

(5) $(1 - 0.78\,L^{12})\,(1 + 0.40\,L)\,b_t = 665.21 + \varepsilon_t$

$R^2 = 0.56$ SER = 1056.15

Q = 16.30

Sample Period: 5/84 – 3/89; T = 59

The Box-Pierce (Q) statistic indicates that the residuals from the model are white noise. The model is appealing in terms of it being a very parsimonious model; also its within sample tracking of monthly deficits is reasonably good, with most of the turning points captured. Figure 2, gives a graph of the in-sample actual and fitted values of monthly deficit.

Equation (5) can be rewritten as

(6) $b_t = 665.21 - 0.40\,b_{t-1} + 0.78\,b_{t-12} + 0.31\,b_{t-13} + \varepsilon_t$

Equation (6) provides a neat interpretation of the adjustment process governing monthly deficits. There is a tendency to reduce the deficit on a month to month basis, indicated by the negative coefficient of b_{t-1}. At the same time, the authorities are not averse to the level of deficit increasing on a yearly basis. This seems to be a reasonable adjustment mechanism as it accounts for the need to contain deficit spending and at the same time makes allowance for the fact that since the series is nominal it is bound to exhibit some trend rate of growth.

Out of Sample Forecasting

Table 2 and Figure 3 present the out of sample forecasts of month-end deficits derived from our model and compares them with the forecasts of the Finance Ministry's B-J model. These are one-month ahead forecasts derived from our model of monthly deficits. In itself, these out of sample forecasts are fairly accurate. Moreover, the forecasts from our model are better than those of the month-end deficit given in the finance ministry's Note.

The root mean squared error (RMSE) of the one-month ahead forecasts of our model is Rs 1,274 crore whereas that of the finance ministry's forecasts is much higher at Rs 4,758 crore. Moreover, in contrast to the forecasts of the finance ministry's B-J model, which are consistently lower than the actual month-end deficits for all twelve months of the forecasting horizon, forecast errors from our model are much more randomly distributed across months. All these suggest that our model is preferable to the B-J model presented in the finance ministry's Technical Note.

The one-month ahead forecasts of month-end deficits, given in Table 2 and Figure 3, are of limited use to the government in 'monitoring' the deficit. As far as monitoring the budget deficit is concerned, the government is interested in knowing, what the year-end or March-end budget deficit is going to be, from different months of a given fiscal year. For example, in, say, August the government has the latest estimate of the actual July-end deficit. Given this, the government wants to know what would the year-end or March-end deficit be. If the forecast of the model is much higher than the government's targeted level of deficit for the year, the government can then take corrective actions to reduce the size of the deficit. This is what monitoring the budget deficit really is.

For monitoring the budget deficit, therefore, one needs the dynamic, multi-period ahead forecasts. For example, if one were to forecast the year-end, i e, March-end, budget deficit in the beginning of May, one needs an eleven month ahead forecast; similarly if one were to forecast it in the beginning of October, one would need a six month ahead forecast and so on. Such forecasts of the year-end deficit for the fiscal year 1989-90, derived from our model, are presented in Table 3 and Figure 4. In Table 3, the dynamic forecast labelled AUG89, for example, gives the forecast for September 1989 to March 1990, as it refers to the dynamic forecast made from the end of August 1989. The horizontal axis of Figure 4 indicates the month in which the forecast for the year-end deficit is made and the vertical axis measures the corresponding forecast of the March-end budget deficit.

The actual year-end deficit in 1989-90 was Rs 11,466 crore. Our model of budget deficits forecasts a year-end deficit of over Rs 10,000 crore as early as in August 1989. From August 1989 onwards, the model predicts a year-end deficit in excess of Rs 10,000 crore and close to Rs 11,000 crore most of the time. This is certainly an impressive forecasting record for the model, considering the fact that the budget deficit (being a small number derived as the difference of two large numbers on government expenditures and receipts) is not a variable easily amenable to econometric modelling and forecasting.

Appendix I

Period	B	b	Period	B	b	Period	B	b
1980.04	-622	-622	1983.08	3115	211	1986.12	8494	965
1980.05	635	1257	1983.09	2613	-502	1987.01	8938	444
1980.06	1668	1033	1983.10	2686	73	1987.02	11432	2494
1980.07	1040	-628	1983.11	3066	380	1987.03	8261	-3171
1980.08	894	-146	1983.12	1569	-1497	1987.04	4204	4204
1980.09	1653	759	1984.01	2467	898	1987.05	4662	458
1980.10	882	-771	1984.02	2343	-124	1987.06	6239	1577
1980.11	1569	687	1984.03	1416	-927	1987.07	5536	-703
1980.12	1530	-39	1984.04	1914	1914	1987.08	6528	992
1981.01	1872	342	1984.05	2854	940	1987.09	7694	1166
1981.02	1726	-146	1984.06	4019	1165	1987.10	5909	-1785
1981.03	2576	850	1984.07	3921	-98	1987.11	7383	1474
1981.04	-12	-12	1984.08	4372	451	1987.12	6704	-679
1981.05	955	967	1984.09	4473	101	1988.01	8128	1424
1981.06	2315	1360	1984.10	3746	-727	1988.02	8445	317
1981.07	1209	-1106	1984.11	4636	890	1988.03	5816	-2629
1981.08	1189	-20	1984.12	4049	-587	1988.04	3754	3754
1981.09	1051	-138	1985.01	4071	22	1988.05	4655	901
1981.10	716	-335	1985.02	4866	795	1988.06	7998	3343
1981.11	915	199	1985.03	3745	-1121	1988.07	8360	362
1981.12	90	-825	1985.04	2092	2092	1988.08	8175	-185
1982.01	208	118	1985.05	2782	690	1988.09	8410	235
1982.02	710	502	1985.06	5610	2828	1988.10	9437	1027
1982.03	1392	682	1985.07	4123	-1487	1988.11	9430	-7
1982.04	726	726	1985.08	5140	1017	1988.12	8644	-786
1982.05	-418	-1144	1985.09	5665	525	1989.01	8883	239
1982.06	-604	-186	1985.10	6622	957	1989.02	9187	304
1982.07	-1269	-665	1985.11	8069	1447	1989.03	5642	-3545
1982.08	5444	6713	1985.12	6814	-1255	1989.04	4628	4628
1982.09	5385	-59	1986.01	7837	1023	1989.05	6777	2149
1982.10	5151	234	1986.02	7988	151	1989.06	9852	3075
1982.11	5983	832	1986.03	5315	-2673	1989.07	11390	1538
1982.12	4469	-1514	1986.04	3251	3251	1989.08	12403	1013
1983.01	4772	303	1986.05	4997	1746	1989.09	12431	28
1983.02	4778	6	1986.06	7132	2135	1989.10	11151	-1280
1983.03	1656	-3122	1986.07	6017	-1115	1989.11	13082	1931
1983.04	0	0	1986.08	6406	389	1989.12	11789	-1293
1983.05	1671	1671	1986.09	5669	-737	1990.01	14504	2715
1983.06	3499	1828	1986.10	7104	1435	1990.02	13908	-596
1983.07	2904	-595	1986.11	7529	425	1990.03	11466	-2442

Notes: B is month-end deficit and b is the corresponding monthly deficit.

III
Summary and Scope for Further Work

In this paper, we estimated a univariate time-series model of monthly deficits and used it to forecast month-end budget deficits for the (out of sample) 1989-90 fiscal year. On the basis of fairly objective criteria, forecasts generated from our autoregressive model appear to be superior to those presented in the finance ministry's Technical Note—the root mean squared error of the out of sample forecasts is lower and the forecast errors, themselves, are much more randomly distributed, as compared to the persistent under prediction by the various models in the Tcehnical Note. Moreover, since the monthly budget deficit, itself, was found to be stationary our model does not run the dangers of overdifferencing as does the B-J model of the Technical Note.

However, the model presented here shares all the limitations of univariate time-series models. Enough scope, therefore, exists for extending the model of budget deficits into a multivariate time-series framework. A first step towards this end may be to model government expenditures and revenues directly, thereby, deriving the budget deficit as the fiscal gap. Such an approach would have two advantages. First, it would allow the modelling of variables which have greater behavioural content. The government can exercise control over its expenditures, but its control over revenues is somewhat limited. Second, a multivariate model would help in addressing the issue of the pattern of financing. Having determined the fiscal gap, the next step is to decide how to finance it, the conventional budget deficit or deficit financing in India being only one of the sources of financing the fiscal gap.

What is needed, in essence, is a fiscal forecasting model. A few fiscal models do exist for India. However, they typically use annual data and, therefore, cannot be used for month-to-month budget deficit monitoring. The data used is also, more often than not, trended. In view of the more recent developments in time-series modelling techniques, there is a need to develop a fiscal forecasting model using not only monthly data but also with adequate attention being paid to the properties of the data and the estimation procedures that these would require.

Appendix II

Implications of Overdifferencing

Let, y_t be a stationary ARIMA (1,0,0) process, represented by

(A1) $y_t = \alpha_1 y_{t-1} + \epsilon_t$; $\quad |\alpha_1| < 1$.

Now, suppose y_t is differenced, to get $\Delta y_t = y_t - y_{t-1}$:

(A2) $\Delta y_t = \alpha_1 (\Delta y)_{t-1} + \epsilon_t - \epsilon_{t-1}$

which implies that y_t is an ARIMA (1,1,1) process. Since $|\alpha_1| < 1$, the process is stationary. However, as the coefficient of ϵ_{t-1} is one, the process is no longer invertible.

Non-invertibility creates problems in that the process cannot be represented as an infinite order AR process, or, in other words, the process depends on a shock to the system at some point in the remote past. This property of non-invertible processes gives rise to inefficient forecasts since the prediction error remains dependent on the initial disturbance to the system, ϵ_0, irrespective of the sample size. Further, as the above example illustrates, overdifferencing also results in the violation of the principle of parsimony - in the given example, starting with a simple AR(1) model, differencing led to an ARIMA(1,1,1) model.

Appendix III

Sample: 1984.05 - 1989.03
Number of observations: 59

Autocorrelations		Partial Autocorrelations			ac	pac
*****	.	*****	.	1	−0.360	−0.360
.	**.	.	* .	2	0.167	0.044
.**	.	. *	.	3	−0.139	−0.076
.	.	. *	.	4	0.007	−0.082
.	* .	.	* .	5	0.069	0.072
.**	.	. *	.	6	−0.127	−0.097
.	* .	.	.	7	0.081	−0.013
. *	.	.	.	8	−0.044	0.014
.**	.	***	.	9	−0.164	−0.236
.	.	.**	.	10	0.025	−0.123
****	.	*****	.	11	−0.299	−0.359
.	*******	.	*****	12	0.562	0.407
***	.	.	**.	13	−0.223	0.143
.	* .	. *	.	14	0.099	−0.085
.	.	.	* .	15	−0.036	0.049
. *	.	. *	.	16	−0.051	−0.073
.	* .	. *	.	17	0.097	−0.046
.	.	.	**.	18	−0.026	0.124
.	.	***	.	19	−0.032	−0.214
.	**.	.	* .	20	0.127	0.094
***	.	.	.	21	−0.216	−0.029
.	* .	.	.	22	0.081	−0.033
***	.	.	* .	23	−0.247	0.076
.	******	.	**.	24	0.428	0.154

Notes

[The views expressed here are the personal views of the author, and are not necessarily those of the Planning Commission, Government of India.]

1 See, Appendix I for a listing of the two series.

2 We deseasonalised monthly deficits using the standard ratio to moving average technique, and found that, indeed, the seasonally adjusted series was negatively autocorrelated at lags 12, 24, . . .

3 A series is said to be stationary if its mean and variance-coveriance matrix is invariant over time. It is easy to show that if a series contains a unit root, its variance becomes a function of time.

4 This period was chosen so as to avoid unnatural outliers.

5 $t_{\hat{\alpha}}$ and $t_{\tilde{\alpha}}$ are t-statistics from (4) and (5) for $\alpha = 1$.

6 We also performed the test proposed by Said and Dickey (1984). The results were the same, namely a rejection of the null hypothesis of a unit root. However, we do not report the statistics here, since simulations have shown that the Phillips-Perron tests have greater power in most cases. The results are available on request.

7 For the Phillips-Perron tests, the serial correlation correction was done for orders 1 to 12. The results were the same, namely, a rejection of the null, in all cases.

8 A simple example in Appendix II illustrates the problems with overdifferencing.

References

Box, G E P and G M Jenkins (1976), *Time Series Analysis: Forecasting and Control*, San Francisco, Holden Day.

Dickey, D A and W A Fuller (1979), 'Distribution of the Estimators for Autoregressive Time Series with a Unit Root', *Journal of the American Statistical Association*, 74, 427-31.

—(1981), 'Likelihood Ratio Statistics for Autoregressive Time Series with a Unit Root', *Econometrica*, 49, 1057-72.

Fuller, W A (1976), *Introduction to Statistical Time Series*, New York, Wiley.

Harvey, A C (1981), *Time Series Models*, Oxford, Philip Allan.

Granger, C W J and P Newbold (1977), *Forecasting Economic Time Series*, New York, Academic Press.

Phillips, P C B and P Perron (1988), 'Testing for a Unit Root in Time Series Regression', *Biometrika*, 75, 335-46.

Said, S E and D A Dickey (1984), 'Testing for Unit Roots in Autoregressive-Moving Average of Unknown Order', *Biometrika*, 71, 599-607.

Wallis, K F (1974), 'Seasonal Adjustment and Relations between Variables', *Journal of the American Statistical Association*, 69, 18-31.